Radiated potentials and fields in isotropic chiral media

1987 J. Phys. A: Math. Gen. 20 4697

(http://iopscience.iop.org/0305-4470/20/14/014)

J. Phys. A: Math. Gen. 20 (1987) 4697-4702. Printed in the UK

Radiated potentials and fields in isotropic chiral media

Vijay K Varadan, Akhlesh Lakhtakia and Vasundara V Varadan

Department of Engineering Science and Mechanics and The Center for the Engineering of Electronic and Acoustic Materials, Pennsylvania State University, University Park, PA 16802, USA

Received 16 February 1987

Abstract. The setting up of a potential theory applicable to isotropic chiral media ($\boldsymbol{D} = \varepsilon \boldsymbol{E} + \beta\varepsilon\nabla\times\boldsymbol{E}$, $\boldsymbol{B} = \mu\boldsymbol{H} + \beta\mu\nabla\times\boldsymbol{H}$) by the specification of vector, as well as scalar, magnetic and electric potentials is reported here. An infinite medium Green dyadic for these potentials is derived and is shown to contain two transverse components as well as a longitudinal one. The vector potentials are trirefringent; however, their longitudinal components do not contribute to the field vectors, which are only birefringent. Moreover, the vector potentials are axial vectors only for achiral media ($\beta = 0$).

1. Introduction

The lack of geometric symmetry between an object and its mirror image is referred to as chirality [1]: the mirror image of such a chiral object cannot be made to coincide with the object itself by any operation involving rotations and/or translations. The most commonly investigated chiral objects are the L- and the D-type stereoisomers so familiar to students of organic chemistry. As a garden variety example, the doubly enantiomorphic sweetner Nutrasweet, patented by GD Searle Company, can occur in four different forms: of these, the taste of L-aspartyl-L-phenylalanine methyl ester is sweet, while that of D-aspartyl-D-phenylalanine methyl ester is bitter; the isomers with the L-D or the D-L configurations are tasteless [2].

The basis for the difference in the physical properties of the mirror conjugates lies in the handedness or the chirality possessed by their microstructures. When an electromagnetic disturbance travels through such a medium, it is forced to adapt to the handedness of the molecules. In other words, linearly polarised plane waves cannot be made to propagate through such a medium, whereas left- and right-circularly polarised plane waves, travelling with different phase velocities, are perfectly acceptable solutions of the vector wave equation for this class of medium [3].

In order to describe the electromagnetic properties of isotropic chiral media, the usual constitutive equations, $\boldsymbol{D} = \varepsilon\boldsymbol{E}$ and $\boldsymbol{B} = \mu\boldsymbol{H}$, are inadequate because they admit to a single phase velocity, which is generally frequency dependent. Instead we must use

$$\boldsymbol{D} = \varepsilon(\boldsymbol{E} + \beta\nabla\times\boldsymbol{E}) \tag{1a}$$

$$\boldsymbol{B} = \mu(\boldsymbol{H} + \beta\nabla\times\boldsymbol{H}) \tag{1b}$$

which are symmetric under time reversality [4] and duality transformations [5]; β is the measure of chirality in (1). The validity of these constitutive equations devolves from studies carried out on optically active molecules [6], as well as from the examination of light propagation in optically active crystals [7]. During the last decade or so, these constitutive relations have been used to compute the scattering responses of bodies possessed with spherical [8] or circular cylindrical [9] geometries. In addition, the authors have explored the interaction of electromagnetic fields with planar achiral-chiral interfaces [10] as well as with non-spherical chiral objects embedded in achiral host media [11].

A systematic study of classical electromagnetic field theory in isotropic chiral media, however, has been lacking, largely because natural chiral (optically active) media have fallen in the province of physical chemists. With modern advances in polymer science, there is reason to believe that artificial chiral dielectrics, active at the millimetre-wave frequencies, may become feasible. With this motivation, the authors have elsewhere [12] obtained the infinite medium Green dyadic, as well as Huyghens' principle for the electric and the magnetic fields in isotropic chiral media, and employed them to set up and investigate a pertinent scattering formalism. As part of their ongoing efforts to understand the electromagnetic waves with chiral media, the authors report here the setting up of a pertinent potential theory by the specification of vector as well as scalar, magnetic and electric potentials. An infinite medium Green dyadic for these potentials will also be derived and expressions for the radiated potentials and fields will be examined.

2. Specification of potentials

Let an isotropic chiral medium occupy the unbounded source-free region $\mathscr{V}$. The third Maxwell's equation, $\nabla\times\boldsymbol{E}=\mathrm{i}\omega\boldsymbol{B}$, can be seen to be completely satisfied by a vector magnetic potential $\boldsymbol{A}$ and a scalar electric potential V, specified by the relations

$$\boldsymbol{H}=\mu^{-1}\nabla\times\boldsymbol{A} \tag{2a}$$

$$\boldsymbol{E}=\mathrm{i}\omega(\boldsymbol{A}+\beta\nabla\times\boldsymbol{A})-\nabla V \tag{2b}$$

subject to the constitutive equations (1). Furthermore,

$$\boldsymbol{D}=\mathrm{i}\omega\varepsilon(\boldsymbol{A}+2\beta\nabla\times\boldsymbol{A}+\beta^2\nabla\times\nabla\times\boldsymbol{A})-\varepsilon\nabla V \tag{3a}$$

$$\boldsymbol{B}=\nabla\times(\boldsymbol{A}+\beta\nabla\times\boldsymbol{A}). \tag{3b}$$

Next, by using (2a) and (3a) in the fourth Maxwell's equation, $\nabla\times\boldsymbol{H}=-\mathrm{i}\omega\boldsymbol{D}$, the equation

$$\nabla^2\boldsymbol{A}+2\gamma^2\beta\nabla\times\boldsymbol{A}+\gamma^2\boldsymbol{A}-\nabla[\nabla\cdot\boldsymbol{A}-\mathrm{i}(\gamma^2/\omega)V]=0 \tag{4}$$

is found to hold in V, where $\gamma^2=k^2(1-k^2\beta^2)^{-1}$ and $k^2=\omega^2\mu\varepsilon$. Provided the gauge condition

$$\mathrm{i}\omega\mu\varepsilon V-(k/\gamma)^2\nabla\cdot\boldsymbol{A}=0 \tag{5}$$

is satisfied, $\boldsymbol{A}$ and $\mathscr{V}$ can be separated from each other and can be shown to satisfy the homogeneous governing differential equations

$$\nabla^2\boldsymbol{A}+2\gamma^2\beta\nabla\times\boldsymbol{A}+\gamma^2\boldsymbol{A}=0 \tag{6a}$$

$$(\nabla^2+\gamma^2)V=0. \tag{6b}$$

In a similar fashion, the fourth Maxwell's equation, $\nabla \times \boldsymbol{H} = -\mathrm{i}\omega \boldsymbol{D}$, can be completely satisfied by a vector electric potential $\boldsymbol{F}$ and a scalar magnetic potential W, which are specified by the relations

$$\boldsymbol{E} = \varepsilon^{-1} \nabla \times \boldsymbol{F} \tag{7a}$$

$$\boldsymbol{H} = -\mathrm{i}\omega(\boldsymbol{F} + \beta \nabla \times \boldsymbol{F}) + \nabla W \tag{7b}$$

$$\boldsymbol{B} = -\mathrm{i}\omega\mu(\boldsymbol{F} + 2\beta \nabla \times \boldsymbol{F} + \beta^2 \nabla \times \nabla \times \boldsymbol{F}) + \mu \nabla W \tag{8a}$$

$$\boldsymbol{D} = \nabla \times (\boldsymbol{F} + \beta \nabla \times \boldsymbol{F}). \tag{8b}$$

Again, provided the gauge condition

$$\mathrm{i}\omega\mu\varepsilon W - (k/\gamma)^2 \nabla \cdot \boldsymbol{F} = 0 \tag{9}$$

holds, $\boldsymbol{F}$ and W can also be shown to satisfy the homogeneous governing differential equations

$$\nabla^2 \boldsymbol{F} + 2\gamma^2 \beta \nabla \times \boldsymbol{F} + \gamma^2 \boldsymbol{F} = 0 \tag{10a}$$

$$(\nabla^2 + \gamma^2) W = 0. \tag{10b}$$

3. Radiated potentials

Let now an electric current density $\boldsymbol{J}$ be impressed on some bounded volume inside $\mathcal{V}$. On using the equations

$$\nabla \times \boldsymbol{E} = \mathrm{i}\omega \boldsymbol{B} \tag{11a}$$

$$\nabla \times \boldsymbol{H} = -\mathrm{i}\omega \boldsymbol{D} + \boldsymbol{J} \tag{11b}$$

it can then be shown that the radiated magnetic potential $\boldsymbol{A}$ in the source-free portion of $\mathcal{V}$ is governed by the relation

$$\nabla^2 \boldsymbol{A} + 2\gamma^2 \beta \nabla \times \boldsymbol{A} + \gamma^2 \boldsymbol{A} = -\mu(\gamma/k)^2 \boldsymbol{J}. \tag{12}$$

Likewise, if a magnetic current density $\boldsymbol{K}$ is radiating, then

$$\nabla \times \boldsymbol{E} = \mathrm{i}\omega \boldsymbol{B} - \boldsymbol{K} \tag{13a}$$

$$\nabla \times \boldsymbol{H} = -\mathrm{i}\omega \boldsymbol{D} \tag{13b}$$

and the radiated electric potential $\boldsymbol{F}$ in the source-free portion of $\mathcal{V}$ has to be computed from the relation

$$\nabla^2 \boldsymbol{F} + 2\gamma^2 \beta \nabla \times \boldsymbol{F} + \gamma^2 \boldsymbol{F} = \varepsilon(\gamma/k)^2 \boldsymbol{K}. \tag{14}$$

The solutions of (12), as well as of (14), require the derivation of a Green dyadic, $\mathfrak{A}(\boldsymbol{r}, \boldsymbol{r}')$, which itself is the solution of the equation

$$[\nabla^2 \mathfrak{J} + \gamma^2 \mathfrak{J} + 2\gamma^2 \beta \nabla \times \mathfrak{J}] \cdot \mathfrak{A}(\boldsymbol{r}, \boldsymbol{r}') = -\mathfrak{J}\delta(\boldsymbol{r} - \boldsymbol{r}') \tag{15}$$

$\mathfrak{J}$ being the unit dyadic. In order to evaluate $\mathfrak{A}(\boldsymbol{r}, \boldsymbol{r}')$, the three-dimensional Fourier transforms

$$\mathfrak{A}(\boldsymbol{r}, \boldsymbol{r}') = (2\pi)^{-3} \iiint_{-\infty}^{\infty} \mathrm{d}^3 p \; \mathfrak{a}(\boldsymbol{p}) \exp[\mathrm{i}\boldsymbol{p} \cdot (\boldsymbol{r} - \boldsymbol{r}')] \tag{16a}$$

$$\delta(\boldsymbol{r} - \boldsymbol{r}') = (2\pi)^{-3} \iiint_{-\infty}^{\infty} \mathrm{d}^3 p \; \exp[\mathrm{i}\boldsymbol{p} \cdot (\boldsymbol{r} - \boldsymbol{r}')] \tag{16b}$$

are utilised in (15), yielding thereby the dyadic relation

$$[(\gamma^2-p^2)\mathfrak{J}+2\mathrm{i}\gamma^2\beta\boldsymbol{p}\times\mathfrak{J}]\cdot\mathfrak{a}(\boldsymbol{p})=-\mathfrak{J}. \tag{17}$$

The solution of (17) can then be found from dyadic algebra [13] and is given as

$$\mathfrak{a}(\boldsymbol{p})=[(p^2-\gamma^2)(p^2-\gamma_1^2)(p^2-\gamma_2^2)]^{-1}(-4\gamma^4\beta^2)\boldsymbol{pp} \\ +[(p^2-\gamma_1^2)(p^2-\gamma_2^2)]^{-1}[(p^2-\gamma^2)\mathfrak{J}+\mathrm{i}2\gamma^2\beta\boldsymbol{p}\times\mathfrak{J}] \tag{18}$$

where $\gamma_1=k(1-k\beta)^{-1}$ and $\gamma_2=\gamma^2/\gamma_1=k(1+k\beta)^{-1}$.

On taking the inverse Fourier transform of (18), vide (16a), it is easy to show that

$$8\pi^3\mathfrak{A}(\boldsymbol{r},\boldsymbol{r}')=\mathfrak{J}K_1+2\gamma^2\beta\nabla\times\mathfrak{J}K_2+4\gamma^4\beta^2\nabla\nabla K_3 \tag{19}$$

in which the integrals

$$K_1=\iiint_{-\infty}^{\infty}\mathrm{d}^3p(p^2-\gamma^2)[(p^2-\gamma_1^2)(p^2-\gamma_2^2)]^{-1}\exp(\mathrm{i}\boldsymbol{p}\cdot\boldsymbol{R}) \\ =(2\pi/\mathrm{i}R)\int_{-\infty}^{\infty}\mathrm{d}p(p^2-\gamma^2)[(p^2-\gamma_1^2)(p^2-\gamma_2^2)]^{-1}p\exp(\mathrm{i}pR) \tag{20a}$$

$$K_2=\iiint_{-\infty}^{\infty}\mathrm{d}^3p[(p^2-\gamma_1^2)(p^2-\gamma_2^2)]^{-1}\exp(\mathrm{i}\boldsymbol{p}\cdot\boldsymbol{R}) \\ =(2\pi/\mathrm{i}R)\int_{-\infty}^{\infty}\mathrm{d}p[(p^2-\gamma_1^2)(p^2-\gamma_2^2)]^{-1}p\exp(\mathrm{i}pR) \tag{20b}$$

$$K_3=\iiint_{-\infty}^{\infty}\mathrm{d}^3p[(p^2-\gamma^2)(p^2-\gamma_1^2)(p^2-\gamma_2^2)]^{-1}\exp(\mathrm{i}\boldsymbol{p}\cdot\boldsymbol{R}) \\ =(2\pi/\mathrm{i}R)\int_{-\infty}^{\infty}\mathrm{d}p[(p^2-\gamma^2)(p^2-\gamma_1^2)(p^2-\gamma_2^2)]^{-1}p\exp(\mathrm{i}pR) \tag{20c}$$

and $\boldsymbol{R}=\boldsymbol{r}-\boldsymbol{r}'$. The evaluation of these integrals must be done in the upper half of the complex plane. Note that the integrands of both K_1 and K_2 contain singularities at $p=\pm\gamma_1,\pm\gamma_2$, while that of K_3 contains yet another singularity at $p=\pm\gamma$; of these six singularities, three have to be excluded because of the chosen time dependence $\exp(-\mathrm{i}\omega t)$. Therefore, after using Cauchy's residue theorem to evaluate the three integrals, the expression for the dyadic $\mathfrak{A}(\boldsymbol{r},\boldsymbol{r}')$ turns out to be

$$\mathfrak{A}(\boldsymbol{r},\boldsymbol{r}')=(k/8\pi\gamma^2)(\gamma_1\mathfrak{J}+\gamma_1^{-1}\nabla\nabla+\nabla\times\mathfrak{J})g(\gamma_1;R) \\ +(k/8\pi\gamma^2)(\gamma_2\mathfrak{J}+\gamma_2^{-1}\nabla\nabla-\nabla\times\mathfrak{J})g(\gamma_2;R)-(1/4\pi\gamma^2)\nabla\nabla g(\gamma;R) \tag{21a}$$

where $g(\sigma;R)=\exp(\mathrm{i}\sigma R)/R$. Note that when $\beta=0$ (achiral media), then (21a) simplifies to the usual expression [14]

$$\mathfrak{A}(\boldsymbol{r},\boldsymbol{r}')=(1/4\pi)\mathfrak{J}g(k;R). \tag{21b}$$

Once the dyadic $\mathfrak{A}(\boldsymbol{r},\boldsymbol{r}')$ has been derived, the radiated vector potentials in the source-free part of $\mathscr{V}$ can be easily evaluated from the integrals

$$\boldsymbol{A}(\boldsymbol{r})=\mu(\gamma/k)^2\iiint\mathrm{d}^3x'\,\mathfrak{A}(\boldsymbol{r},\boldsymbol{r}')\cdot\boldsymbol{J}(\boldsymbol{r}') \tag{22a}$$

$$\boldsymbol{F}(\boldsymbol{r})=-\varepsilon(\gamma/k)^2\iiint\mathrm{d}^3x'\,\mathfrak{A}(\boldsymbol{r},\boldsymbol{r}')\cdot\boldsymbol{K}(\boldsymbol{r}') \tag{22b}$$

in which the integrations are performed over the volumes containing the respective source current densities. The corresponding scalar potentials can then be obtained from the gauge conditions (5) and (9), if needed.

4. Radiated fields

The radiated fields $\boldsymbol{E}$ and $\boldsymbol{H}$ themselves satisfy the inhomogeneous equations

$$\nabla\times\nabla\times\boldsymbol{E}-2\gamma^2\beta\nabla\times\boldsymbol{E}-\gamma^2\boldsymbol{E}=\mathrm{i}\omega\mu(\gamma/k)^2(\boldsymbol{J}+\beta\nabla\times\boldsymbol{J})-(\gamma/k)^2(\nabla\times\boldsymbol{K}) \tag{23a}$$

$$\nabla\times\nabla\times\boldsymbol{H}-2\gamma^2\beta\nabla\times\boldsymbol{H}-\gamma^2\boldsymbol{H}=\mathrm{i}\omega\varepsilon(\gamma/k)^2(\boldsymbol{K}+\beta\nabla\times\boldsymbol{K})+(\gamma/k)^2(\nabla\times\boldsymbol{J}) \tag{23b}$$

the solutions of which require the evaluation of the Green dyadics, $\mathfrak{G}(\boldsymbol{r},\boldsymbol{r}')$ and $\mathfrak{H}(\boldsymbol{r},\boldsymbol{r}')$, which, respectively, satisfy the dyadic relations

$$(-\nabla\nabla+\nabla^2\mathfrak{I}+\gamma^2\mathfrak{I}+2\gamma^2\beta\nabla\times\mathfrak{I})\cdot\mathfrak{G}(\boldsymbol{r},\boldsymbol{r}')=-\mathfrak{I}\delta(\boldsymbol{r}-\boldsymbol{r}') \tag{24a}$$

$$(-\nabla\nabla+\nabla^2\mathfrak{I}+\gamma^2\mathfrak{I}+2\gamma^2\beta\nabla\times\mathfrak{I})\cdot\mathfrak{H}(\boldsymbol{r},\boldsymbol{r}')=-\nabla\times\mathfrak{I}\delta(\boldsymbol{r}-\boldsymbol{r}'). \tag{24b}$$

The solution procedure for (24a) is the same as for (15), and the final result can be stated as [12]

$$\begin{aligned}\mathfrak{G}(\boldsymbol{r},\boldsymbol{r}')&=(k/8\pi\gamma^2)(\gamma_1\mathfrak{I}+\gamma_1^{-1}\nabla\nabla+\nabla\times\mathfrak{I})g(\gamma_1;R)\\&\quad+(k/8\pi\gamma^2)(\gamma_2\mathfrak{I}+\gamma_2^{-1}\nabla\nabla-\nabla\times\mathfrak{I})g(\gamma_2;R).\end{aligned} \tag{25a}$$

On the other hand, $\mathfrak{H}(\boldsymbol{r},\boldsymbol{r}')$ need not be explicitly calculated: inspection of (24b) with respect to (24a) leads to the establishment of the identity

$$\mathfrak{H}(\boldsymbol{r},\boldsymbol{r}')=\nabla\times\mathfrak{G}(\boldsymbol{r},\boldsymbol{r}'). \tag{25b}$$

Furthermore, while, by comparing (21a) with (25a), the relation

$$\mathfrak{A}(\boldsymbol{r},\boldsymbol{r}')=\mathfrak{G}(\boldsymbol{r},\boldsymbol{r}')-(1/4\pi\gamma^2)\nabla\nabla g(\gamma;R) \tag{26a}$$

can be obtained, it should be noted that, because the second term on the right-hand side of (26a) is irrotational,

$$\nabla\times\mathfrak{A}(\boldsymbol{r},\boldsymbol{r}')=\nabla\times\mathfrak{G}(\boldsymbol{r},\boldsymbol{r}'). \tag{26b}$$

Whether from (2), (7) and (22), or from (23) and (25), the radiation fields in the source-free part of $\mathscr{V}$ can be computed via the relations

$$\begin{aligned}\boldsymbol{E}(\boldsymbol{r})&=\mathrm{i}\omega\mu(\gamma/k)^2\iiint \mathrm{d}^3x'(\mathfrak{I}+\beta\nabla\times\mathfrak{I})\cdot\mathfrak{G}(\boldsymbol{r},\boldsymbol{r}')\cdot\boldsymbol{J}(\boldsymbol{r}')\\&\quad-(\gamma/k)^2\iiint \mathrm{d}^3x'(\nabla\times\mathfrak{I})\cdot\mathfrak{G}(\boldsymbol{r},\boldsymbol{r}')\cdot\boldsymbol{K}(\boldsymbol{r}')\end{aligned} \tag{27a}$$

$$\begin{aligned}\boldsymbol{H}(\boldsymbol{r})&=\mathrm{i}\omega\varepsilon(\gamma/k)^2\iiint \mathrm{d}^3x'(\mathfrak{I}+\beta\nabla\times\mathfrak{I})\cdot\mathfrak{G}(\boldsymbol{r},\boldsymbol{r}')\cdot\boldsymbol{K}(\boldsymbol{r}')\\&\quad+(\gamma/k)^2\iiint \mathrm{d}^3x'(\nabla\times\mathfrak{I})\cdot\mathfrak{G}(\boldsymbol{r},\boldsymbol{r}')\cdot\boldsymbol{J}(\boldsymbol{r}')\end{aligned} \tag{27b}$$

in which the integrations are performed over the volumes containing the source current densities.

5. Discussion

The immediate consequence of the expression (25a) for $\mathfrak{G}(\boldsymbol{r}, \boldsymbol{r}')$ is that the electromagnetic field vectors must exhibit birefringence, vide the wavenumbers γ_1 and γ_2, in an isotropic chiral medium. Waves traversing with a phase velocity ω/γ_1 are left-circularly polarised (LCP), while the wavenumber γ_2 is associated with the right-circularly polarised (RCP) waves. In an unbounded chiral medium, the LCP and the RCP waves can propagate without interfering with each other, one of them having a slower phase velocity than the other. But when a wave of either polarisation encounters a boundary, mode conversion takes place; the scattered field then consists, in general, of waves of both polarisations [8-11]. Unattenuated propagation of both the LCP and the RCP waves occurs provided both k and β are real. If $\mathrm{Im}(k)+|k|^2\,\mathrm{Im}(\beta)=0$, γ_1 is real and LCP waves traverse the chiral medium without losing any energy. On the other hand, if $\mathrm{Im}(k)-|k|^2\,\mathrm{Im}(\beta)=0$, γ_2 is real and RCP waves propagate in the medium without suffering any attenuation. It need hardly be mentioned that a linearly polarised plane wave cannot travel in an isotropic chiral medium, except in the limiting case of $\beta=0$.

Whereas the electromagnetic field vectors simply exhibit birefringence, it turns out to be quite surprising to observe that the potential vectors, $\boldsymbol{A}$ and $\boldsymbol{F}$, are trirefringent. From (21a), $\mathfrak{A}$ is comprised of three components, each having a different wavenumber—γ_1, γ_2 and $\gamma=(\gamma_1\gamma_2)^{1/2}$—but all three being of order $\mathrm{O}(1/R)$ in the limit $R\to\infty$. The first two components of $\mathfrak{A}$ are solenoidal, leading to the identity (26b); the third one, however, having phase velocity ω/γ, is purely longitudinal and does not contribute to the radiated $\boldsymbol{E}$ and $\boldsymbol{H}$, which are solenoidal whatever the value of β. Incidentally, γ is a valid wavenumber for the potentials, as shown by the homogeneous differential equations (6b) and (10b) to which, respectively, the scalar potentials V and W must conform. Finally, $\boldsymbol{A}$ and $\boldsymbol{F}$ are not axial vectors for $\beta\neq 0$; consequently, and because the solenoidal part of $\mathfrak{A}$ precisely equals $\mathfrak{G}$, the calculation of the radiated electromagnetic fields in isotropic chiral media through $\boldsymbol{A}$ and $\boldsymbol{F}$ is as simple or difficult as a direct calculation through $\mathfrak{G}$.

References

[1] Jaggard D L, Mickelson A R and Papas C H 1978 *Appl. Phys.* **18** 211
[2] Goodman M 1985 *Biopolymers* **24** 137
[3] Post E J 1962 *Formal Structure of Electromagnetics* (Amsterdam: North-Holland)
[4] Satten R A 1958 *J. Chem. Phys.* **28** 742
[5] Silverman M P 1985 *Lett. Nuovo Cimento* **43** 378
[6] Eyring H, Walter J and Kimball G E 1944 *Quantum Chemistry* (New York: Wiley)
[7] Fedorov F I 1959 *Opt. Spectrosk.* **6** 49
[8] Bohren C F 1974 *Chem. Phys. Lett.* **29** 458
[9] Bohren C F 1978 *J. Colloid Interface Sci.* **66** 105
[10] Lakhtakia A, Varadan V V and Varadan V K 1986 *IEEE Trans. Electromag. Compat.* **EC-28** 90
[11] Lakhtakia A, Varadan V K and Varadan V V 1985 *Appl. Opt.* **24** 4146
[12] Lakhtakia A, Varadan V V and Varadan V K 1986 *J. Opt. Soc. Am.* A submitted
[13] Chen H C 1983 *Theory of Electromagnetic Waves* (New York: McGraw-Hill)
[14] Jackson J D 1975 *Classical Electromagnetics* (New York: Wiley)